Online Appendix for *A theory of social programs, legitimacy and citizen cooperation with the state*

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Appendix 1: Formal Proofs

Proof of Proposition 1. In order to show this result, I show that an increase in the parameter α leads to a decrease in citizen cooperation. From Equation (6), by the implicit function theorem

$$\frac{ds^*}{d\alpha} = -\frac{\frac{\partial f}{\partial s}}{\alpha \frac{\partial^2 f(s^*)}{\partial s^2}}$$

which is clearly negative. Moreover, from Equation (5) and the implicit function theorem, m^* is not affected by changes in α. Then, by the fact that citizen cooperation is increasing in the amount of goods and services provided by the state, citizen cooperation goes down. ■

Proof of Proposition 2. From Equation (5) and the implicit function theorem we have that $\frac{\partial m^*}{\partial \lambda} = 0$. From Equation (6) and the implicit function theorem we have that

$$\frac{ds^*}{d\lambda} = \frac{V}{2\gamma\alpha \frac{\partial f^2(s^*)}{\partial s^2}}$$

which is clearly positive. From Equation (3) and the above results we then have that

$$\frac{di^*}{d\lambda} = \frac{s^*}{\gamma} + \frac{\lambda}{\gamma}\frac{ds^*}{d\lambda} > 0.$$

■

Proof of Proposition 3. Note that from the first order condition of the the state's choice of an optimal level of s, given in Equation (6), and from the implicit function theorem, the following holds:

$$\frac{ds^*}{dV} = \frac{\lambda}{2\gamma\alpha\frac{\partial^2 f(s^*)}{\partial s^2}}$$

which is clearly positive. On the other hand by the implicit function theorem and Equation (5), we have the following

$$\frac{dm^*}{dV} = \frac{\frac{1}{2} - \frac{\theta}{2\gamma}}{\beta\frac{\partial g^2(m^*)}{\partial m^2}}$$

The denominator is clearly positive and by Assumption 2, so is the numerator, making the whole expression positive. Hence an increase in the value of the dismantling the criminal organization produces an increase in both the use of hard tools, m, and soft tools, s.

We already know that citizen cooperation increases with s and decreases with m. Taking the total derivative of (3) with respect to V we have the following

$$\frac{di^*}{dV} = \frac{\lambda}{\gamma}\frac{ds^*}{dV} - \frac{\theta}{\gamma}\frac{dm^*}{dV}$$

Unless we make further assumptions we cannot sign this equation. To show that it is possible that cooperation decreases, suppose $\lambda \to 0$, the the first term will be infinitesimally small. Cooperation will then clearly go down as V increases. Conversely, to show that cooperation may increase, suppose that $\theta \to 0$; in this case, the second term will be infinitesimally small and cooperation will clearly increase as a result of an increase in V. ■

Proof of Proposition 4.

Proof. From Equation (10) and the implicit function theorem we have that

$$\frac{dm_1^*}{d\lambda} = -\delta \frac{\frac{1}{2} - \frac{\theta}{2\gamma}}{\beta \frac{\partial^2 g}{\partial m^2}} \frac{\partial \hat{U}_S}{\partial \lambda}$$

which given our assumptions takes the opposite sign of $\frac{\partial \hat{U}_S}{\partial \lambda}$.

On the other hand have that

$$\frac{ds_1^*}{d\lambda} = \frac{V(1+\delta) - \delta \hat{U}_S - \delta \lambda \frac{\partial \hat{U}_S}{\partial \lambda}}{2\gamma\alpha \frac{\partial^2 f}{\partial s_1^2}}$$

Note that in order to calculate $\frac{\partial \hat{U}_S}{\partial \lambda}$ we can directly apply the envelope theorem to Equation (4) evaluated at the optimal m_2^* and s_2^*. It follows that

$$\frac{\partial U_S}{\partial \lambda} = \frac{s_2^* V}{2\gamma} > 0$$

Hence, since by Assumption 2 $\theta < \gamma$, clearly $\frac{dm_1^*}{d\lambda} < 0$. Moreover, to calculate the sign of $\frac{ds_1^*}{d\lambda}$ note that from inspection of the state's utility function that $\hat{U}_S < V$, naturally then $\delta \hat{U}_S < V$. Also note that s_2^* is capped at 1 and by assumption it must be the case that since $W > 0$, $\lambda < 2\gamma$. It follows that $\delta \frac{s_2^* V}{\gamma} < \delta V$. Hence $\frac{ds_1^*}{d\lambda} > 0$. ■

Proof of Proposition 5.

From Equation (9) we can take the total derivative of i_1^* with respect to λ, which is

$$\frac{di_1^*}{d\lambda} = \frac{s_1^*}{\gamma} + \frac{\lambda}{\gamma} \frac{V(1+\delta) - \delta \hat{U}_S - \delta \lambda \frac{\partial \hat{U}_S}{\partial \lambda}}{2\gamma\alpha \frac{\partial^2 f}{\partial s_1^2}} + \frac{\theta}{\gamma} \delta \frac{\frac{1}{2} - \frac{\theta}{2\gamma}}{\beta \frac{\partial g}{\partial m}} \frac{\partial \hat{U}_S}{\partial \lambda} - \frac{\delta}{2\gamma} \frac{\partial \hat{U}_C}{\partial \lambda} \tag{A.1}$$

where $\frac{\partial \hat{U}_C}{\partial \lambda} = s_2^* i_2^* + (\lambda i_2^* + \kappa) \frac{ds_2^*}{d\lambda}$. The value of $\frac{ds_2^*}{d\lambda} = \frac{\lambda}{2\gamma\alpha \frac{\partial^2 f(s_2^*)}{\partial s_2^2}}$ as established in Proposition 2. While the first three terms of Equation (A.1) are clearly positive, the fourth is negative and depends on the value of κ. To prove the result, note that as $\kappa \to \infty$ the whole expression becomes negative; conversely as either $\delta \to 0$ or $\kappa \to 0$, the expression is clearly positive. By the continuity of Equation (A.1) on κ and the intermediate value theorem, there exists a κ at which the expression changes sign. ■.

In order to prove Lemmas 1 and and 2 it is useful to first prove the following Lemma.

Lemma 3. *The determinant of* $Z \equiv \frac{\partial(F_1,F_2,F_3)}{\partial(e,m,s)} < 0$

Proof.

$$Z = \begin{pmatrix} \frac{-W+2\theta m-2\lambda s}{4\gamma e^3}2Y & \frac{-\theta}{2\gamma e^2}Y & \frac{\lambda}{2\gamma e^2}Y \\ \frac{\theta V}{2\gamma e^2} & -\beta\frac{\partial^2 g}{\partial m^2} & 0 \\ \frac{-\lambda}{2\gamma e^2}V & 0 & -\alpha\frac{\partial^2 f}{\partial s^2} \end{pmatrix}$$

Its determinant is given by

$$\frac{-W+2\theta m-2\lambda s}{4\gamma e^3}2Y\big(\beta\frac{\partial^2 g}{\partial m^2}\big)\big(\alpha\frac{\partial^2 f}{\partial s^2}\big)-\big(\frac{-\theta}{2\gamma e^2}Y\big)\big(\frac{\theta V}{\gamma e^2}\big)\big(-\alpha\frac{\partial^2 f}{\partial s^2}\big)-\big(\frac{\lambda}{2\gamma e^2}Y\big)\big(\frac{\lambda}{2\gamma e^2}V\big)\big(\beta\frac{\partial^2 g}{\partial m^2}\big)$$

Of which all terms are negative. ■

We can now prove Lemma 1 and Lemma 2.

Proof of Lemma 1.

To prove Lemma 1 we must show that $\frac{de^*}{d\lambda} > 0$. Note that by the implicit function theorem

$$\frac{de^*}{d\lambda} = -\frac{det\frac{\partial(F_1,F_2,F_3)}{\partial(\lambda,m,s)}}{det\frac{\partial(F_1,F_2,F_3)}{\partial(e,m,s)}}$$

.

Note that the denominator is the determinant of Z, which by Lemma 3 is negative. Hence we only need to show that $det\frac{\partial(F_1,F_2,F_3)}{\partial(\lambda,m,s)} > 0$.

$$\frac{\partial(F_1,F_2,F_3)}{\partial(\lambda,m,s)} = \begin{pmatrix} \frac{s}{2\gamma e^2}Y & -\frac{\theta}{2\gamma e^2}Y & \frac{\lambda}{2\gamma e^2}Y \\ 0 & -\beta\frac{\partial^2 g}{\partial m^2} & 0 \\ \frac{1}{2\gamma e}V & 0 & -\alpha\frac{\partial^2 f}{\partial s^2} \end{pmatrix}$$

It's determinant is given by:

$$\frac{s}{2\gamma e^2}Y(\beta\frac{\partial^2 g}{\partial m^2})(\alpha\frac{\partial^2 f}{\partial s^2}) + (\frac{\lambda}{2\gamma e^2}Y)(\beta\frac{\partial^2 g}{\partial m^2})(\frac{1}{2\gamma e})$$

All of its terms are positive, and hence $\frac{de^*}{d\lambda} > 0$. ■

Proof of Lemma 2.

In order to show that the use of hard tools is increasing, we must show that

$$\frac{dm^*}{d\lambda} = -\frac{det\frac{\partial(F_1,F_2,F_3)}{\partial(e,\lambda,s)}}{det\frac{\partial(F_1,F_2,F_3)}{(e,m,s)}} > 0$$

.

Given the results from Lemma 3, in order to do so, we must show that $det\frac{\partial(F_1,F_2,F_3)}{\partial(e,\lambda,s)} > 0$. In order to do so, note the following:

$$\frac{\partial(F_1,F_2,F_3)}{\partial(e,\lambda,s)} = \begin{pmatrix} \frac{-W+2\theta m-2\lambda s}{4\gamma e^3}2Y & \frac{s}{2\gamma e^2}Y & \frac{\lambda}{2\gamma e^2}Y \\ \frac{\theta V}{2\gamma e^2} & 0 & 0 \\ \frac{-\lambda}{2\gamma e^2}V & \frac{1}{2\gamma e} & -\alpha\frac{\partial^2 f}{\partial s^2} \end{pmatrix}$$

Its determinant is given by

$$\big(\frac{s}{2\gamma e^2}Y\big)\big(\frac{\theta V}{2\gamma e^2}\big)\big(\alpha\frac{\partial^2 f}{\partial s^2}\big) + \big(\frac{\lambda}{2\gamma e^2}Y\big)\big(\frac{\theta V}{2\gamma e^2}\big)\big(\frac{1}{2\gamma e}\big)$$

All of its terms are positive and hence $\frac{dm^*}{d\lambda} > 0$.

In order to show that the use of soft tools is increasing, we must show that

$$\frac{ds^*}{d\lambda} = -\frac{det\frac{\partial(F_1,F_2,F_3)}{\partial(e,m,\lambda)}}{det\frac{\partial(F_1,F_2,F_3)}{\partial(e,m,s)}} > 0$$

In order to do so, given the results from Lemma 3, we must show that $det\frac{\partial(F_1,F_2,F_3)}{\partial(e,m,\lambda)} >$ 0. In order to do so, note the following:

$$\frac{\partial(F_1,F_2,F_3)}{\partial(e,m,\lambda)} = \begin{pmatrix} \frac{-W+2\theta m-2\lambda s}{4\gamma e^3}2Y & -\frac{\theta}{2\gamma e^2}Y & \frac{s}{2\gamma e^2}Y \\ \frac{\theta V}{2\gamma e^2} & -\beta\frac{\partial^2 g}{\partial m^2} & 0 \\ \frac{-\lambda}{2\gamma e^2}V & 0 & \frac{V}{2\gamma e} \end{pmatrix}$$

Its determinant is given by

$$\Big(\frac{W-2\theta m+2\lambda s}{4\gamma e^3}2YV\Big)\Big(\beta\frac{\partial^2 g}{\partial m^2}\Big)\Big(\frac{1}{2\gamma e}\Big)+\Big(\frac{\theta}{2\gamma e^2}Y\Big)\Big(\frac{\theta V}{2\gamma e^2}\Big)\Big(\frac{1}{2\gamma e}\Big)-\Big(\frac{s}{2\gamma e^2}Y\Big)\Big(\frac{\lambda}{2\gamma e^2}V\Big)\Big(\beta\frac{\partial^2 g}{\partial m^2}\Big)$$

This reduces to

$$\Big(\frac{2W-4\theta m+2\lambda s}{8\gamma^2 e^4}YV\Big)\Big(\beta\frac{\partial^2 g}{\partial m^2}\Big)+\Big(\frac{\theta}{2\gamma e^2}Y\Big)\Big(\frac{\theta V}{2\gamma e^2}\Big)\Big(\frac{1}{2\gamma e}\Big)$$

All of its terms are positive and hence $\frac{ds^*}{d\lambda} > 0$. ■

Proof of Proposition 6.

From Equation (16) take the derivative of i^* with respect to λ. This yields

$$\frac{di^*}{d\lambda} = \frac{\big(-2\theta\frac{dm}{d\lambda}+2s+2\lambda\frac{ds}{d\lambda}\big)2\gamma e-2\gamma\frac{de}{d\lambda}\big(W-2\theta m+2\lambda s\big)}{4\gamma^2 e^2}$$

By Assumption 1 and Lemma 1, the second term of the numerator is negative, but without further assumptions we cannot sign the first term, leaving the sign of the whole expression ambiguous. In particular, note that if $V \to 0$, since in equilibrium aid will very small and tend to 0 itself, and increasing the value of λ will not change that, the first term will tend to 0 and the expression will be negative. If on the other hand, $\beta \to 0$, then by Lemma 1, $\frac{de^*}{d\lambda} \to 0$ and if $\theta \to 0$, the negative part of the first term disappears, rendering the whole expression positive. ■

Appendix 2: Hard Tools and Cooperation as Complements

I this appendix I show the case in which $p(m,i) = mi$, meaning that the use of hard tools and citizen cooperation with the state are complements in determining the probability with which the state defeats the insurgency. Then the citizen's utility function is given by:

$$U_C(m,s,i) = miW - \frac{i^2}{2} + \lambda si - \omega mi$$

In turn, the state's utility function is given by:

$$U_S(m,s,i) = miV - \alpha f(s) - \beta g(m)$$

I show that in this case, a larger marginal legitimacy value of the provision of aid, is related with a higher provision of aid but also with a higher use of hard tools, which may lead to a increase or a decrease in citizen cooperation. In order to do this, I assume that if $W > \theta$, then $\beta \frac{\partial^2 g}{\partial m^2} > \frac{2(w-\theta)V}{\gamma}$ for all m. This guarantees that the costs of using hard tools are always increasing faster than the benefits.[1] Finally I assume that $\alpha \frac{\partial^2 f}{\partial s^2}(\beta \frac{\partial^2 g}{\partial m^2} - \frac{2(w-\theta)V}{\gamma}) > \frac{\lambda^2 V^2}{\gamma^2}$, which means that the strategic complementarities between m and s are not too large as to completely offset the costs of increasing their use.

Proposition 2A. *In places where the marginal impact on legitimacy from the use of soft tools is higher, states will provide higher levels of goods and services, will use more hard tools, and may either enjoy higher or lower levels of cooperation by the citizenry.*

Proof. Directly from the first order condition of the citizen's utility function, we can see that

$$i^*(m,s) = \frac{mW + \lambda s - \theta m}{\gamma}$$

[1]Note that I allow for the possibility of $W \leq \theta$.

Substituting into the state's utility function, we obtain:

$$U_S(m,s) = \frac{m^2(W-\theta)+\lambda ms}{\gamma}V - \alpha s - \beta g(m)$$

From this we obtain the following first order conditions:

$$G_1 = \frac{\partial U_S}{\partial m} = \frac{2m(W-\theta)+\lambda s}{\gamma}V - \beta\frac{\partial g}{\partial m} = 0$$

$$G_2 = \frac{\lambda m}{\gamma}V - \alpha\frac{\partial f}{\partial s} = 0$$

This constitutes a system of implicit equations. By the implicit function theorem

$$\frac{dm^*}{d\lambda} = -\frac{det\frac{\partial(G_1,G_2)}{\partial(\lambda,s)}}{det\frac{\partial(G_1,G_2)}{\partial(m,s)}}$$

In the above equation

$$\frac{\partial(G_1,G_2)}{\partial(\lambda,s)} = \begin{pmatrix} \frac{SV}{\gamma} & \frac{\lambda V}{\gamma} \\ \frac{mV}{\gamma} & -\alpha\frac{\partial^2 f}{\partial s^2} \end{pmatrix}$$

which has a negative determinant. It is also the case that

$$\frac{\partial(G_1,G_2)}{\partial(m,s)} = \begin{pmatrix} \frac{2(W-\theta)V}{\gamma} - \beta\frac{\partial^2 s}{\partial m^2} & \frac{\lambda V}{\gamma} \\ \frac{\lambda V}{\gamma} & -\alpha\frac{\partial^2 f}{\partial s^2} \end{pmatrix}$$

which given the assumptions above has a positive determinant. Hence $\frac{dm^*}{d\lambda} > 0$. Moreover, given this implicit system,

$$\frac{ds^*}{d\lambda} = -\frac{det\frac{\partial(G_1,G_2)}{\partial(m,\lambda)}}{det\frac{\partial(G_1,G_2)}{\partial(m,s)}}$$

in which

$$\frac{\partial(G_1,G_2)}{\partial(m,\lambda)} = \begin{pmatrix} \frac{2(W-\theta)V}{\gamma} - \beta\frac{\partial^2 s}{\partial m^2} & \frac{SV}{\gamma} \\ \frac{\lambda V}{\gamma} & \frac{mV}{\gamma} \end{pmatrix}$$

which given the assumptions above has a negative determinant, and thus $\frac{ds^*}{d\lambda} > 0$. Going back to $i^*(m,s)$, note that

$$\frac{di^*}{d\lambda} = \frac{dm^*}{d\lambda}\frac{W-\theta}{\gamma} + \frac{s}{\gamma} + \frac{\lambda}{\gamma}\frac{ds^*}{d\lambda}$$

If $W \geq \theta$, then citizen cooperation unambiguously increases as the marginal legitimacy impact of aid increases; however if $W < \theta$, then we cannot unambiguously sign the equation. In particular if $\theta \to \infty$, then the whole expression will be negative. ■

This result shows that if the citizen has a particularly large distaste for the use of hard tools, then an increase in the marginal effect of legitimacy, which will lead the authority to use more hard tools will backfire and result in less citizen cooperation with the state. A similar result arises, when the state's value of defeating the insurgency increases. I show this in the following proposition.

Proposition 3A. *If the state's value for dismantling a criminal organization goes up, citizen cooperation may either increase or decrease.*

Proof. From the state's first order conditions above, by the implicit function theorem we can derive that

$$\frac{dm^*}{dV} = -\frac{det\frac{\partial(G_1,G_2)}{\partial(V,s)}}{det\frac{\partial(G_1,G_2)}{\partial(m,s)}}$$

in which

$$\frac{\partial(G_1,G_2)}{\partial(V,s)} = \begin{pmatrix} \frac{2(m(W-\theta)+\lambda s}{\gamma} & \frac{\lambda V}{\gamma} \\ \frac{\lambda m}{\gamma} & -\alpha\frac{\partial^2 f}{\partial s^2} \end{pmatrix}$$

which if $W \geq \theta$ clearly has a negative determinant. Paired with the fact that $det\frac{\partial(G_1,G_2)}{\partial(m,s)} > 0$, by the implicit function theorem, $\frac{dm^*}{dV} > 0$. If $W < \theta$ then it is possible that either $det\frac{\partial(G_1,G_2)}{\partial(V,s)} < 0$, if $\lambda s > 2m(W-\theta)$, but otherwise $det\frac{\partial(G_1,G_2)}{\partial(V,s)} > 0$ and thus $\frac{dm^*}{dV} < 0$.

In order to determine the effect of a change in V on the use of soft tools, we can calculate

$$\frac{ds^*}{dV} = -\frac{det\frac{\partial(G_1,G_2)}{\partial(m,V)}}{det\frac{\partial(G_1,G_2)}{\partial(m,s)}}$$

in which

$$\frac{\partial(G_1,G_2)}{\partial(m,V)} = \begin{pmatrix} \frac{2(W-\theta)V}{\gamma} - \beta\frac{\partial^2 s}{\partial m^2} & \frac{2m(W-\theta)+\lambda s}{\gamma} \\ \frac{\lambda V}{\gamma} & \frac{\lambda m}{\gamma} \end{pmatrix}$$

The determinant of which is negative, regardless of the whether W or θ are larger. Hence, unambiguously $\frac{ds^*}{dV} > 0$. In order to assess the effect on citizen cooperation we can calculate from the citizen's first order conditions the following

$$\frac{di^*}{d\lambda} = \frac{dm^*}{dV}\frac{W-\theta}{\gamma} + \frac{\lambda}{\gamma}\frac{ds^*}{dV}$$

If $W \geq \theta$ then unambiguously all the terms are greater or equal than zero and citizen cooperation increases as the state's value for defeating the insurgency increases. However if $W < \theta$ and $\lambda \to 0$ then $\frac{dm^*}{dV} > 0$ and the second term of the last equation will tend to zero, then citizen cooperation with the state will decrease, proving the statement.■

Intuitively what this last result shows is that if m and i are complements in the probability of the state defeating the insurgency, then an increase in the state's valuation of defeating the insurgency may lead to an increase in both m and s, but may lead to a decrease in i if the citizen's cost of cooperating with the state increases too much as a result of the increase in m and the benefit of cooperating with the state as a result of an increase in i is particularly low.

Appendix 3: Model with incomplete information

In what follows I discuss an extension to the model developed in the first section. In particular I analyze whether some of the main results are robust to having a different modeling approach to legitimacy in which by giving a transfer the state reveals information about itself. Suppose now that legitimacy is a characteristic inherent to the authority: more *virtuous* states derive a higher utility from providing goods and services to the citizen than more *venal* states. Formally, I operationalize this as the state's utility function having an additive benefit from using soft tools, λs. Moreover suppose that a state's legitimacy, λ is private information and has some continuous distribution function over the interval $[\lambda_L, \lambda_H]$. The state's utility function is then given by:

$$U_S(m, s, i) = \frac{m+i}{2}V - \alpha f(s) - \beta g(m) + \lambda s$$

I assume that $\alpha f'(s) > \lambda$ for any s. The citizen's utility function in turn is given by:

$$U_C(m, s, i) = \frac{m+i}{2}W - \frac{i^2}{2}\gamma - \theta mi + \lambda i$$

Since the citizen does not know the value of λ, we need to analyze her expected utility given the state's provision of goods and services:

$$\mathbb{E}[U_C(\cdot|s)] = \frac{m+i}{2}W - \frac{i^2}{2}\gamma - \theta mi + \int_{\lambda_L}^{\lambda_H} p(l|s)li\ dl$$

where l is just the variable of integration. I only solve for the separating Perfect Bayesian Equilibrium of this game.[2] Note that under separation $s(\lambda)$ is one-to-one and hence its inverse $L(s)$ exists. Hence, under separation the following must hold for any given pairs of λ and s

$$p(\lambda|s) = \begin{cases} 1 & \text{if} \quad s(\lambda) = s \\ 0 & \text{if} \quad s(\lambda) \neq s \end{cases}$$

and thus under separation the citizen's expected utility becomes

$$\mathbb{E}[U_C(\cdot|s)] = \frac{m+i}{2}W - \frac{i^2}{2}\gamma - \theta mi + L(s)i$$

The citizen's first order condition then implies that

$$i^* = \frac{W}{2\gamma} + \frac{-\theta m + L(s)}{\gamma}$$

[2]In the pooling equilibrium, no type of authority gives a transfer and the citizen does not learn any new information about the authority.

Clearly from Equation (A.2) we can see that $\frac{di}{ds}$ takes the same sign as $\frac{\partial L}{\partial s}$. Substituting this result into the state's utility function, its first order conditions are

$$\frac{\partial U_S}{\partial m} = \frac{1}{2}V(1 - \frac{\theta}{\gamma}) - \beta\frac{\partial g}{\partial m} = 0 \tag{A.3}$$

$$\frac{\partial U_S}{\partial s} = \frac{1}{2\gamma}\frac{\partial L}{\partial s}V - \alpha\frac{\partial f}{\partial s} + \lambda = 0 \tag{A.4}$$

From the assumptions and the above first order conditions and we can derive the following result.

Lemma 4. *In any separating equilibrium an increase in the provision of goods and services implies that the authority is more legitimate.*

Proof. By assumption, from Equation(A.4) the sum of the last two terms is negative, and hence $\frac{\partial L}{\partial s} > 0$, otherwise the equation would be negative and we would have a pooling equilibrium, in which all types of states would have $s = 0$. ∎.

However it still remains to be shown that indeed more legitimate states provide more goods and services. From Equation (A.4) and the implicit function theorem we have the following Proposition.

Proposition 7. *More legitimate states provide more goods and services.*

Proof. If Equation (A.4) identifies a maximum, then by the implicit function theorem, the sign of $\frac{ds}{d\lambda}$ is the same as that of $\frac{\partial^2 U_S}{\partial s \partial \lambda} = 1 > 0$. It follows that its inverse is also increasing. ∎

The satisfaction of Equation (A.4) for all λ is a necessary and sufficient condition for the existence of a separating equilibrium (Mailath, 1987). By sequential rationality, it must be the case that the lowest type of state, that with $\lambda = \lambda_L$, sets $s = 0$, since if they set any higher level, they would enjoy the same level of cooperation from the citizenry and would be worse off by spending more; this fact paired with Equation (A.4) establishes a unique separating equilibrium. Lemma 4 and Proposition 7 together establish that there exists a separating equilibrium if and only if more legitimate states use more soft tools.

Similarly to the results with complete information, an increase in the valuation of winning the conflict for the state has uncertain implications.

Proposition 8. *An increase in the state's valuation of winning the conflict increases both the use of hard tools and the provision of goods and services, and thus, ceteris paribus, citizen cooperation, for low values of θ, may go up or, for high values of λ, may go down.*

Proof. From Equations (A.3) and (A.4) and the implicit function theorem, it can be seen that $\frac{ds^*}{dV} > 0$ and $\frac{dm^*}{dV} > 0$. Taking the total derivative of (19) with respect to V we have

$$\frac{di}{dV} = -\frac{\theta}{\gamma}\frac{dm^*}{dV} + \frac{dL}{ds}\frac{ds}{dV}$$

which has an uncertain sign, since the first term is negative and the second positive. However, if $\theta \to 0$, then the expression will be clearly positive. Similarly, if $\lambda \to \alpha\frac{df}{ds}$, then $\frac{dL}{ds} \to 0$, and the whole expression will be negative. ∎

Proposition 8 then shows that even if we model legitimacy in a slightly different way, there are still conditions under which a state that provides more social programs and good and services to the citizenry, might face lower levels of cooperation against an insurgency. In particular, if the citizen does not have a particular distaste for the use of hard tools, low θ, citizen cooperation will increase as a result of an increase in the state's value of increasing the conflict, V. On the contrary, states with high levels of legitimacy, meaning a high λ, may find it difficult to increase cooperation as a result of an increase in V. The reason is that these kinds of states will already enjoy high levels of cooperation and will thus find it difficult to increase it even more.